Solution by the PROPOSER.

We have given $a_1{}^n+a_2{}^n+a_3{}^n+\ldots+a_r{}^n=A^n\ldots(1)$.

Dividing by A^n, $\left(\frac{a_1}{A}\right)^n+\left(\frac{a_2}{A}\right)^n+\left(\frac{a_3}{A}\right)^n+\ldots\left(\frac{a_r}{A}\right)^n=1\ldots(2)$.

Since the sum of the terms in the left member of (2) is 1, each term is <1. Hence each of the fractions

$$\frac{a_1}{A},\quad \frac{a_2}{A},\quad \frac{a_3}{A},\ \ldots,\ \frac{a_r}{A}$$

is a proper fraction. Then, if in (2) we substitute m for n, we shall have

$$\left(\frac{a_1}{A}\right)^m+\left(\frac{a_2}{A}\right)^m+\left(\frac{a_3}{A}\right)^m+\ldots+\left(\frac{a_r}{A}\right)^m> \text{ or } <1,$$

according as $m<$ or $>n$, as is clearly evident. Multiplying by A^m, $a_1{}^m+a_2{}^m+a_3{}^m+\ldots+a_r{}^m>$ or $<A^m$, according as $m<$ or $>n$.

271. Proposed by GEORGE H. HALLETT, Ph. D., Assistant Professor of Mathematics in The University of Pennsylvania, Philadelphia, Pa.

Find the simplest integral form of the sum $y(y-1)\ldots(y-x)+2y(2y-1)\ldots(2y-x)+\ldots+zy(zy-1)\ldots(zy-x)$.

Dr. Zerr obtains $\frac{1}{x!}\sum_{r=y}^{r=zy}\int_0^1\left[\log\left(\frac{1}{u}\right)\right]^r du$ as the sum of the series. This does not satisfy the requirements of the problem for the reason that the sum is to be in integral form. ED. F.

272. Proposed by PROF. R. D. CARMICHAEL, Anniston, Ala.

Prove that the relations $x=\frac{ar+bs}{\lambda}=\frac{as-br}{\mu}=\frac{a\lambda-b\mu}{r}=\frac{a\mu+b\lambda}{s}$ between the finite real quantities $x, a, b, r, s, \lambda, \mu$ requires that $x^2=a^2+b^2$.

I. Solution by the PROPOSER.

These relations make the determinant

$$\Delta\equiv\begin{vmatrix}\lambda x+i\mu x & (ar+bs)+i(as-br)\\ (a\lambda-b\mu)+i(a\mu+b\lambda) & rx+isx\end{vmatrix},\ (i=\sqrt{-1}),$$

necessarily $=0$; for its columns are identical. Dividing the first column by $\lambda+i\mu$ and the second by $r+is$, we have

$$\Delta\equiv(\lambda+i\mu)(r+is)\begin{vmatrix}x & a-ib\\ a+ib & x\end{vmatrix}=0.$$

Hence,

$$\begin{vmatrix}x & a-ib\\ a+ib & x\end{vmatrix}=0;\ \text{or } x^2=a^2+b^2.$$

II. Solution by J. SCHEFFER, A. M., Hagerstown, Md., and A. H. HOLMES, Brunswick, Me.

By clearing of fractions, adding the four resulting equations and solving for $x-a$, we have

$$x-a=b(\lambda-\mu-\nu+s)/(\lambda+\mu+\nu+s)\ldots(1).$$

Also, after the equations are cleared of fractions, by subtracting the sum of the second and third from the sum of the first and fourth, and solving for $x+a$, we have

$$x+a=b(\lambda+\mu+\nu+s)/(\lambda-\nu-\mu+s)\ldots(2).$$

Multiplying (1) by (2), we have $x^2-a^2=b^2$ or $x^2=a^2+b^2$.

Also solved by G. W. Greenwood and G. B. M. Zerr.

Professor Greenwood furnished two solutions, the first of which made use of the vanishing of the determinant of the coefficients of λ, μ, ν, s in order that the four equations be consistent. The second solution was obtained by finding the values of μ/λ, ν/λ, s/λ in any three of the four equations and substituting them in the remaining equation.

Dr. Zerr solved each of the four equations for $x\lambda$, $x\mu$, $x\nu$, and xs, respectively, and added the square of the resulting equations. By dividing this last equation by $\Sigma\lambda^2$, x^2 is immediately found.

Professors Zerr and Greenwood also sent in solutions of 271.

GEOMETRY.

300. Proposed by J. J. QUINN, Ph. D., Scottdale, Pa.

Trisect an angle by means of a tractrix.

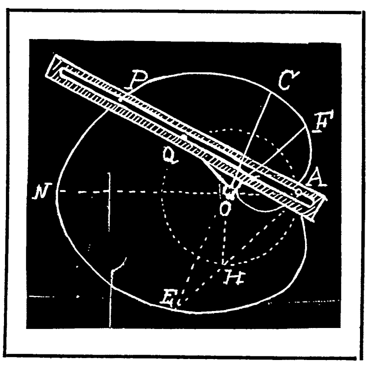
No solution has been received.

301. Proposed by PROF. R. D. CARMICHAEL, Anniston, Ala.

Apply the locus of $r=a(1+2\cos\theta)$ to the trisection of an angle. Describe the curve by continuous motion.

Solution by B. F. FINKEL, Ph. D., Drury College, Springfield, Mo.

To describe the curve by continuous motion, we may construct an instrument of the following description:



Take a bar AP and cut a path, indicated in the figure by the continuous white line, equal in length to four times the radius OA of the circle. Fasten the radius OQ at the middle point, Q, of the bar AP by means of a joint. Fasten the other end, O, of the radius at O. At P, a distance from Q equal to OQ, insert a pen point. Then if the path in the bar is set over a fixed pin at A, at a distance from O equal to OQ, the point P will describe the curve as the bar is made to move so as to keep A in the path of the bar.